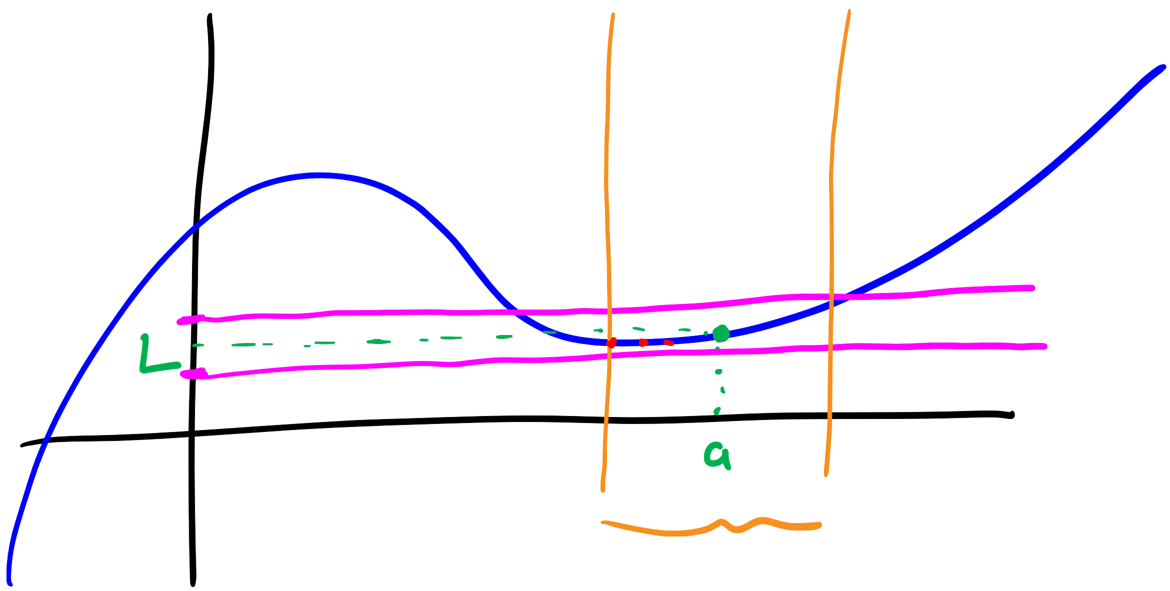
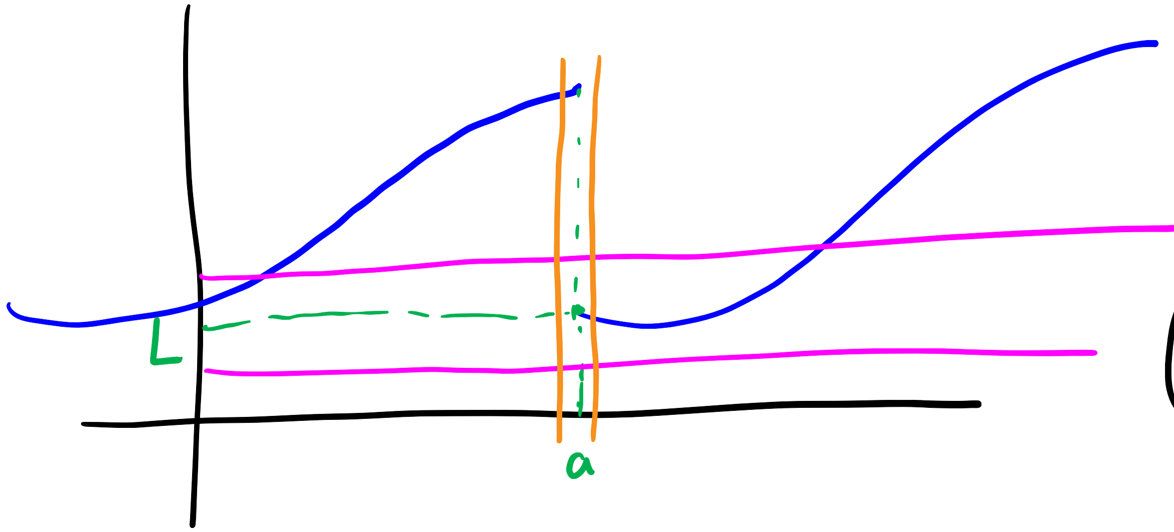
Limits

<u>Def</u>: Suppose $f(x)$ is defined near $x=a$. Then we write $\lim_{x\to a} f(x) = L$ if we can make the values of f arbitrarily close to L by restricting x to be sufficiently close to a, but not equal to a.





Limit Does Not exist $\left(\lim_{x\to a} f(x)\right)$

Ex: (1) $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$

x	$f(x)$
1.9	3.9
1.99	3.99
1.999	3.999

$\downarrow$ 4

x	$f(x)$
2.1	4.1
2.01	4.01
2.001	4.001

$\downarrow$ 4

$\lim_{x\to 2^-} \frac{x^2-4}{x-2} = 4$

"from the left"

$\lim_{x\to 2^+} \frac{x^2-4}{x-2} = 4$

"from the right"

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(2) $\lim_{x\to 0} \frac{4}{1+e^{1/x}}$ Does Not Exist (DNE)

left

x	$f(x)$
-0.1	3.99982
-0.01	4.00000
-0.001	4.00000

$\downarrow$ 4

right

x	$f(x)$
0.1	0.00018
0.01	0.00000
0.001	0.00000

$\downarrow$ 0

$$\lim_{x\to 0^-} \frac{4}{1+e^{1/x}} = 4$$

$$\lim_{x\to 0^+} \frac{4}{1+e^{1/x}} = 0$$

③ $$\lim_{x\to 0^+} \sin\left(\frac{\pi}{x}\right)$$

x	$\sin\left(\frac{\pi}{x}\right)$
0.1	$\sin\left(\frac{\pi}{0.1}\right) = \sin(10\pi) = 0$
0.01	$\sin\left(\frac{\pi}{0.01}\right) = \sin(100\pi) = 0$
0.001	$\sin(1000\pi) = 0$
0.0001	$\sin(10000\pi) = 0$

$\downarrow$

0

$$\lim_{x\to 0^-} f(x) = \frac{1}{2} \qquad \lim_{x\to 0^+} f(x) = 4$$

$$\lim_{x\to 0} f(x) \text{ DNE} \qquad \lim_{x\to 2} f(x) = \frac{1}{2}$$

$$\lim_{x\to 4} f(x) = \infty$$